MATHEMATICS

Paper 9709/01
Paper 1

General comments

The paper produced results very similar to the past two years and it was rare to find that candidates had had insufficient time to complete the paper The standards of algebra and numeracy were generally good, though use of $a + b = \frac{c}{d} \Rightarrow a + bd = c$ was an error that affected several parts of questions, particularly **Question 11**. Particular points to note were the lack of understanding of the word magnitude in vectors and the failure by many candidates to recognise the significance of the word *exact*. The candidates should realise that they are being requested to evaluate without the use of a calculator and that decimal checks are not acceptable.

Comments on specific questions

Question 1

This proved to be a successful starting question for all but the weaker candidates who failed to recognise the need to differentiate. The differentiation was usually accurate and generally marks were only lost through failure to cope with the negative signs involved.

Answer: 12.

Question 2

This was poorly answered with many candidates showing a complete lack of understanding of how to cope with the double angle. Most candidates attempted to express the given equation in the form tan $2x = k$, though a disturbing number stated that $\frac{\sin 2x}{\cos 2x} = \frac{\sin x}{\cos x}$. Of those obtaining tan $2x = -3$, many obtained only one value for $2x$ and consequently x, whilst a high proportion obtained a value for x and then looked at quadrants, instead of looking at quadrants before dividing by two. Among the other methods seen were the use of the t formulae, expressing the original equation in the form $R\sin(2x + \alpha)$ and expanding both sin $2x$ and cos $2x$ and recognising the resulting equation as a quadratic in $\frac{\sin x}{\cos x}$.

Answers: 54.2º, 144.2º.

Question 3

This was generally a source of high marks with some weaker candidates even evaluating each amount for the first 11 years. Although a significant number incorrectly assumed that the situation was modelled by an arithmetic progression, most used the appropriate formula for a geometric progression accurately. The most common error was to fail to realise that in 2011, n equals 11, not 10.

Answers: **(i)** $8140; **(ii)** $71 000.

Question 4

There were many very good attempts with most candidates using powers of 2 and the binomial co correctly. Marks were generally only lost for arithmetic or algebraic slips. A small minority elected to re the '2' from the bracket, but at least a half of these took $(2+ax)^n$ as $2\left(1+\frac{ax}{2}\right)^n$. Some errors came fro misuse of the negative sign in finding *a*, but the most common error came from using $(ax)^2$ as ax^2.

Answers: $n = 5$, $a = -\frac{1}{2}$, $b = 20$.

Question 5

This proved to be one of the more successfully answered questions and the standard of algebra was sound. Candidates electing to eliminate *x* were generally more successful, since the manipulation of $(4x + 6)^2$ in eliminating *y* was too often seen as $16x^2 + 36$. The vast majority of candidates gained method marks for the solution of their quadratic and for finding the distance between their two points.

Answer: 3.75.

Question 6

Attempts varied considerably. Candidates must be aware that an instruction to find a value **exactly** means that decimal answers obtained from a calculator will lose the accuracy marks available. A very significant number failed to realise the need to use '$\cos 30° = \frac{1}{2}\sqrt{3}$'. Similarly, in finding expressions for angle *CAB* or for finding the length of *AC*, decimal checking that a candidate's decimal answer is the same as the given expression, will gain only the method marks available. Candidates also need to be aware that asking for a proof implies that all necessary working will be shown. Stating that $(4 + 3\sqrt{3})^2 + 9 = 52 + 24\sqrt{3}$ is insufficient for the final accuracy mark.

Answer: **(i)** $3\sqrt{3}$.

Question 7

It was pleasing to note that even when candidates failed to cope with part **(i)**, they invariably proceeded to parts **(ii)** and **(iii)** and marks scored were high. In part **(i)**, at least a third of all candidates failed to realise the need to use trigonometry in triangle *OAT* (or *OBT*). Parts **(ii)** and **(iii)** were more successful, with formulae for arc length and sector area being accurately applied. The only real problem came in the final part when many candidates struggled to recognise that the kite *OABT* comprised two right-angled triangles.

*Answers***:** **(ii)** 47.3 cm; **(iii)** 50.9 (±0.1) cm².

Question 8

Part **(i)** was poorly answered. A majority of candidates failed to obtain a correct answer for $\overrightarrow{OD}$. The most successful attempts were written down directly, presumably by considering '4 units along the **i** direction, 4 units along the **j** direction and 5 units along the **k** direction'. The fact that at least a third of all attempts ignored the request to find the magnitude, but then used the magnitude correctly in part **(ii)**, suggested that the meaning of the word was not fully understood. Part **(ii)** was however very well answered with the vast majority of candidates identifying $\overrightarrow{OB}$ and gaining the method marks for applying the scalar product correctly.

Answers: **(i)** 4**i** + 4**j** + 5**k**, 7.55 m; **(ii)** 43.7º (or 0.763 radians).

Question 9

Apart from a small minority of solutions in which candidates took the equation of a curve to be the same as the equation of the tangent, this was well answered. In part **(i)** most candidates correctly used $m_1m_2 = -1$ and obtained the equation of the normal, and consequently the mid-point of *QR.* In part **(ii)**, candidates knew to integrate, but answers were affected by three main errors. Many candidates faile[illegible] $\frac{4}{\sqrt{6-2x}}$ as $4(6-2x)^{-\frac{1}{2}}$, others failed to divide by −2 (the differential of the bracket), and a very s[illegible] number ignored the constant of integration.

Answers: **(i)** (8.5, 4.25); **(ii)** $y = 16 - 4\sqrt{6-2x}$.

Question 10

Attempts varied considerably on this question, though there were many excellent solutions. In part **(i)**, candidates usually realised the need to differentiate, to set the differential to 0 and then to solve for *x*. Surprisingly, many candidates left the ‘+*k*’ in the differential and met irresolvable problems in solving for *x*. Of those obtaining values for *x* at the turning points, many failed to realise the need to put $y = 0$ alongside the positive value of *x* to evaluate *k*. Surprisingly only a minority of attempts managed to find the coordinates of the maximum point. Similarly in part **(iii)**, only a small proportion of candidates realised that the curve was decreasing for all values of *x* between the maximum and minimum points. Part **(iv)** was well done, with most candidates realising the need to integrate and performing the integration accurately.

Answers: **(i)** 27; **(ii)** (−1, 32); **(iii)** $-1 < x < 3$; **(iv)** 33.75.

Question 11

Apart from the second request in part **(i)**, the question was very well answered, and generally a source of high marks. In part **(i)**, nearly all candidates formed a quadratic equation in *x* and recognised that using ‘$b^2 - 4ac = 0$’ would lead to two values for *k*. Unfortunately, algebraic errors, particularly in obtaining *a*, *b* and *c* meant that the quadratic equation was usually incorrect. Rewriting $k - x = \frac{9}{x+2}$ as $k - x(x+2) = 9$ or using $b^2 - 4ac$ as ‘< 0’ or ‘> 0’ were both common errors. Very few candidates realised the need to return to the original quadratic with their values of *k* before the repeated root could be obtained. Parts **(ii)** and **(iii)** were very well answered and were usually correct. It was pleasing to note that very rarely was gf used instead of fg in part **(ii)**.

*Answers***:** **(i)** $k = 4$ or -8, $x = 1$ or -5; **(ii)** 7; **(iii)** $\frac{9-2x}{x}$.

MATHEMATICS

Paper 9709/02

Paper 2

General comments

Many candidates had been well prepared and demonstrated considerable finesse in what they did. However, there were also many candidates who had little knowledge of the syllabus, were very weak at manipulating procedures and had a poor grasp of basic calculus techniques. In general, presentation was reasonably good, and there was no sign of candidates running out of time. Certain questions, or parts thereof, proved highly popular and were well attempted, e.g. **Question 1**, **Question 2(ii)** and **Question 4**. However, poor responses were the norm in **Question 6(iii)** and **Question 7(iv)**.

Comments on specific questions

Question 1

Most candidates opted to square each side and consider the resulting quadratic inequality. However, a significant member of candidates squared only the left-hand side of the original inequality, and many who squared correctly on both sides later made numerical errors. The majority of candidates scored the initial two marks, but then incorrectly deduced that $2 < x < 5$, despite the fact that $x = 0$ clearly satisfied the original inequality. Those who relied on simply removing the inequality's modulus signs invariably obtained only the solution $x = 5$.

Answer: $x < 2$, $x > 5$.

Question 2

(i) Although the majority of the solutions were excellent, many candidates had no idea of how to begin, due largely to the errors $\cos(x + 30°) = \cos x + \cos 30°$, $\sin(x + 60°) = \sin x + \sin 60°$.

(ii) Virtually all candidates used the results of part **(i)** to deduce that $x = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and obtained the correct answer.

Answer: **(ii)** 54.7.

Question 3

A surprisingly high number of candidates could not obtain a correct value for $\frac{dy}{dx}$. Others began well but obtained only a solution in 0 ø x ø $\frac{\pi}{2}$. Often solutions were presented in degrees, rather than radians. When seeking to decide if the stationary points were of the maximum or minimum type, several candidates confused the relationship between the sign of the second derivative and the nature of the point.

Answers: maximum at $x = \frac{1}{6}\pi$, minimum at $x = \frac{5}{6}\pi$.

Question 4

(i) Aside from a few candidates setting p(+2) = 0 instead of p(–2) = 0, the principal problem solving two simultaneous equations for *a* and *b*. This largely was a case of one or more errors.

(ii) A substantial minority obtained a quadratic factor, by dividing the given cubic by (x – 1) or by (x + 2), but made no attempt to factorise their quadratic. What was required was to divide p(x) by (x – 1)(x + 2).

Answers: **(i)** $a = 2$, $b = 3$; **(ii)** $2x + 1$.

Question 5

(i) Many candidates made no discernible attempt at any differentiation. Others could not differentiate xy and/or y^2. Several did not appreciate that the derivative of a constant equals zero. Those who correctly differentiated sometimes believed that $\frac{dy}{dx} = 1$ when the tangent is parallel to the x-axis, or that $y = 0$ was appropriate.

(ii) Even candidates who struggled with part **(i)** were usually able, in part **(ii),** to set $y = -3x$ in the quadratic relationship although many obtained only one of the two required points. Others set $y = 0$ or $x = 0$ in the given x-y relationship in the stem of the question.

Answers: **(ii)** (1, –3), (–1, 3)

Question 6

(i) Sketches of $y = 9e^{-2x}$ were generally poor, with many asymptotic to the y-axis , and others not even hyperbolic in form.

(ii) There was a widespread reluctance to define a function of the form $(x - 9e^{-2x})$ and to consider its values at $x = 1, 2$. Arguments were often very vague and errors abounded, e.g. looking at the sign of $(1 - 9e^{-4})$, instead of considering $(2 - 9e^{-4})$, when $x = 2$.

(iii) There were very few good attempts. Hardly anyone realized that as $n \to \infty$, so x_n and x_{n+1} tend to a limiting value X such that $X = \frac{1}{2}(\ln 9 - \ln X)$ and hence that $X = 9e^{-2X}$.

(iv) The iteration was invariably well done, though many answers were given to 4 or 3 decimal places rather than to 2. Often not enough iterations were pursued; 6 iterations were strictly required before the final answer could legitimately be found.

Answer: **(iv)** $x = 1.07$.

Question 7

(i) The factor 2 was often missing from the numerator as candidates generally did not use the chain rule.

(ii) A factor $\frac{1}{2}$ was missing from most answers, with no use of the chain rule in reverse.

(iii) A surprisingly high proportion of candidates wrongly started the sign of the reminder correctly, despite accurate long division, or gave both the quotient and the remainder as being equal to $2x$.

(iv) Almost no solutions were correct, largely because the result of part **(iii)** was not used to convert the integrand into the form $2x + 1 - \frac{3}{2x+3}$. There was a wide range of results of performing an indefinite integration, with none based an any recognizable calculus results.

*Answers***:** **(i)** $\frac{2}{2x+3}$; **(iii)** quotient = $2x + 1$, remainder = –3.

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Paper 9709/03

Paper 3

General comments

The standard of work by candidates varied greatly and resulted in a wide spread of marks from zero to full marks. The paper appeared to be accessible to fully prepared candidates and no question seemed to be of unreasonable difficulty. All questions discriminated well and there appeared to be sufficient time for candidates to attempt them. The questions on which candidates generally scored highly were **Question 4** (trigonometry) and **Question 9** (partial fractions). Those which were least well answered were **Question 2** (inequality) and **Question 10** (vector geometry).

The presentation of work continues to be generally satisfactory. However there are some candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if centres could discourage the practice. There are also candidates who do not always show sufficient steps or reasoning to justify their answers. In particular, when the answer to a problem is given in the question paper, for example as in **Question 3**, Examiners penalize the omission of essential working.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a very good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Nearly all candidates took logarithms of both sides of a relation, though some were insecure when they manipulated logarithms. For example, the error of treating $\frac{\ln a}{\ln b}$ as $\ln\left(\frac{a}{b}\right)$, or *vice versa*, was seen quite frequently. Some mistakenly took $3(4^{-y})$ to be 12^{-y}, and others, having established a correct expression for $-y$, made errors handling the minus sign. There were many different approaches to this problem and a number of alternative forms of the correct answer were seen.

Answer: $\frac{\ln 4 - \ln x}{\ln 3}$.

Question 2

Completely sound responses to the problem of solving $2x > |x - 1|$ were rare and usually came from candidates who made use of a sketch of $y = 2x$ and $y = |x - 1|$ on a single diagram.

Candidates who worked with the non-modular quadratic inequality obtained by squaring both sides seemed unaware of the limitations of the approach. With an inequality such as this, involving a linear expression and the modulus of a linear expression, the method cannot be safely relied on to do more than identify possible critical values of the original inequality. In this particular problem one of the two possibilities found by squaring turns out to be critical for the original inequality. Only a few candidates realised this and went on to solve the problem correctly. For a further illustration of the weakness of the method, consider the inequality

$x > |2x + 1|$. This has no critical values and is false for all values of x. The correspond[illegible] inequality $x^2 > (2x+1)^2$ has two critical values and is true for $-1 < x < -\frac{1}{3}$.

Candidates who worked with non-modular linear inequalities almost always ignored the conditions und[illegible] which these inequalities were defined. For example, the solution of $2x > 1 - x$, the form of the original inequality when x ø 1, was taken to be $x > \frac{1}{3}$ rather than $\frac{1}{3} < x$ ø 1. Similarly, in the case when $x > 1$, the inequality is equivalent to $2x > x - 1$. Here almost all candidates took the solution to be $x > -1$ rather than $x > 1$.

Answer: $x > \frac{1}{3}$.

Question 3

This question was quite well answered. The method for dealing with the parametric equations of a curve seemed to be understood and accurate use was made of the double angle formulae. Errors usually arose at the beginning when the derivatives of x and y with respect to θ were being found.

Question 4

Part **(i)** was generally answered well though α was not always stated to the required degree of accuracy. In part **(ii)** most candidates went on to obtain the solution 126.9° but few found the other solution 20.6°. Many candidates obtained and rejected 380.6°, which is not in the required interval, but failed to realise that this meant that 20.6° would be a solution.

Answers: **(i)** $R = 25$, $\alpha = 73.74°$; **(ii)** 20.6°, 126.9°.

Question 5

There were few correct solutions to part **(i)** as most candidates assumed the equation was of the form $\frac{dx}{dt} = k(x - 250)$ rather than $\frac{dx}{dt} = kx - 25$. In part **(ii)** the variables were usually separated correctly and the integration of the differential equation was generally well done. Sometimes the constant of integration was introduced at the wrong time, for example, after exponentiating the integrals of both sides. In attempting to obtain an expression for x in terms of t, errors in the manipulation of logarithms and in exponentiation were quite common.

Answer: **(ii)** $x = 250(3\,e^{0.1t} + 1)$

Question 6

(i) This was very poorly answered. The graph of $y = \cot x$ rarely had $x = 0$ as asymptote, or passed through $(\frac{1}{2}\pi, 0)$. When a recognizable graph of $y = 1 + e^x$ was drawn it was quite often shown to pass through (0, 1) rather than (0, 2). Having drawn correct graphs with one intersection in the given range some candidates failed to state or indicate the required conclusion about roots.

(ii) Some seemed to believe that a statement involving ‘positive’ and ‘negative’ was sufficient, without any reference to there being a change of sign, or even to the function under consideration. However, others did clearly state the function they were considering and evaluated numerical values as required, before stating what the change of sign meant.

(iii) This was generally well answered though some appeared to believe that $\tan^{-1}(x)$ is the reciprocal of $\tan x$.

(iv) The request for the result of each iteration to be given to 4 decimal places seems to h d candidates' solutions, though some failed to give the final answer to 2 decimal places. calculated in degree mode obtained 33.5692 as the first iterate. Since an earlier part ha that the desired root lay between 0.5 and 1 the size of this iterate should have signalle something was wrong. However such candidates invariably went on iterating and wasted valu time on fruitless work.

Answer: **(iv)** 0.61.

Question 7

In part **(i)** most candidates formed the complex conjugate correctly, but errors in plotting 2 + i, 2 – i and 4 on an Argand diagram were surprisingly frequent. Even if the points were plotted correctly, the geometrical relationship between *O*, *A*, *B* and *C* was not often satisfactorily described. Part **(ii)** was well answered but there were very few satisfactory proofs in part **(iii)**.

*Answers***:** **(i)** *OACB* is a rhombus (or equivalent); **(ii)** $\frac{3}{5}+\frac{4}{5}\mathrm{i}$.

Question 8

Part **(i)** was fairly well answered. Most candidates obtained a correct form of the derivative but errors arose in the algebraic work involved in finding the *x*-coordinate of the stationary point.

There were some good short solutions to part **(ii)**. However there were also some very poor answers. Though nearly all attempts used $u = \ln x$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = x^{\frac{1}{2}}$, Examiners found that the formula for integration by parts was quite frequently misapplied. When it is correctly applied it leads to a further integral of $\frac{2}{3}x^{\frac{3}{2}}.\frac{1}{x}$. Rather than simplify this product to $\frac{2}{3}x^{\frac{1}{2}}$ before integrating, many candidates took the integral to be the product of the integral of $\frac{2}{3}x^{\frac{3}{2}}$ and the integral of $\frac{1}{x}$.

Answers: **(i)** e^{-2}; **(ii)** 4.28.

Question 9

This question was very well answered. In part **(i)** most candidates set out with a correct form of partial fractions and had a sound method for finding the unknown constants.

In part **(ii)** the expansion of $(1+x^2)^{-1}$ was usually handled correctly. The expansion of $(2-x)^{-1}$ proved more difficult. Errors were made in converting it to the form $k\left(1-\frac{x}{2}\right)^{-1}$ and errors of sign occurred in the terms of the expansion, especially in the x^3 term.

Answers: **(i)** $\frac{2}{2-x}+\frac{2x+4}{1+x^2}$; **(ii)** $5+\frac{5}{2}x-\frac{15}{4}x^2-\frac{15}{8}x^3$.

Question 10

Part **(i)** was usually correctly answered. Some mistakenly took the direction of the line *l* to be that of *AB*.

In part **(ii)**, though many different successful methods were seen, Examiners also encountered much confused and confusing work. The most common approach began by solving a parametric equation obtained by setting the scalar product of two vectors equal to zero. But the two vectors were not always the right ones. Thus some answers gave the perpendicular from *O* to *l*, rather than from *B* to *l*. Other answers appeared to be finding a perpendicular to *AB*. Some of these misconceptions might have been avoided if candidates had planned their solution with the aid of a sketch. Finally it was not uncommon for a candidate to obtain the correct value of the parameter of *N* and find *BN* correctly but omit to find the pos[illegible]f *N*.

Part **(iii)** was found less difficult and there were some confident solutions. Sign errors and small alg[illegible] slips were the main sources of error.

Candidates should take care to check their working in these vector questions. In particular, errors of sign seem to occur when applying the distributive law, when subtracting vector components, or simply copying a vector in component form from one part of an answer to another. For example, in this question the vector $\begin{pmatrix}3\\-1\\-4\end{pmatrix}$ was sometimes miscopied as $\begin{pmatrix}3\\-1\\4\end{pmatrix}$.

Answers: **(i)** $\mathbf{r} = \begin{pmatrix}-1\\3\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\-4\end{pmatrix}$; **(ii)** $\begin{pmatrix}5\\1\\-3\end{pmatrix}$; **(iii)** $7x - 11y + 8z = 0$.

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Paper 4

General comments

A disappointingly large number of candidates were not suitably prepared for examination and scored very low marks. Almost all of the candidates not in this category scored all or nearly all of the available marks in the first three questions. Almost all also scored at least some marks in **Question 4** and in **Question 6**, but **Question 5** and **Question 7** were generally very poorly attempted.

This pattern of performance is indicative of the intended incline of difficulty of the questions. However the incline proved very steep at the upper end; in respect of each of **Question 5** and **Question 7** there were very many candidates who scored no marks at all.

Comments on specific questions

Question 1

This question was very well attempted with most candidates scoring full marks.

Answers: **(i)** 0.5; **(ii)** 300.

Question 2

This question was well attempted and many candidates scored full marks. Among those candidates who recognised the need to use calculus, the only common mistake was to solve $\frac{dv}{dt} = 0$ instead of $v = 0$ in part **(i)**.

A significant minority of candidates attempted to answer the question using constant acceleration formulae, and scored no marks.

Answers: **(i)** 100 s; **(ii)** 1670 m.

Question 3

This question was well attempted. Among candidates who used basically correct methods the most common errors were of accuracy. The use of prematurely approximated values of trigonometric ratios was very prevalent; even one significant figure approximations were not uncommon.

Answers: 28.3, 44.8.

Question 4

Parts **(i)** and **(ii)** were generally well attempted. The most common errors were answers of 0.5 m from $\frac{1}{2}(7-5)\times 0.5$ in part **(i)(b)**, and 1.67 ms^{-2} from $\frac{7-5}{1.2}$ in part **(ii)**.

Part **(iii)** was less well attempted with very many candidates omitting the weight of the stone when using Newton's second law. Another common error was to use $a = 4$ instead of $a = -4$.

Answers: **(i)(a)** 2.4 m, **(b)** 3 m; **(ii)** 4 ms^{-2}; **(iii)** 0.05 kg.

Question 5

The vast majority of candidates were unable to show that the tension in the string is 4 N. Such ca[ndidates] fall into three categories in respect of their approach to the remaining parts of the question.

Candidates in the first category accepted the given value of 4 N for the tension, and used this to find the mass of *Q* and the mass of *P*. They then used their values for the masses in answering part **(iii)**. Such candidates scored all or nearly all of the remaining 7 marks for the question.

Candidates in the second category chose instead to use a value different from 4 N for the tension. Inevitably such candidates failed to score the single mark for m_Q. Because it was rare for candidates to state explicitly that $F = T$, such candidates almost always failed to score either the method mark or the accuracy mark when an incorrect value for F was used in part **(ii)**. Two of the accuracy marks in part **(iii)** were out of reach of those who used incorrect masses. In order to score accuracy marks candidates must be encouraged to use the given data, including in this case $T = 4$, even if this data is at odds with their own calculation.
Candidates in the third category misunderstood the data, believing that the force of magnitude $4\sqrt{2}$ N is an externally applied force. In almost all cases candidates in this category were unable to score any marks for the question.

Another misunderstanding arising in this question relates to the particle attached to *Q*. A significant minority of candidates treated the particle of mass 0.1 kg as a replacement for *Q* instead of an addition to *Q*.

Answers: **(i)** 0.4 kg; **(ii)** 0.5 kg; **(iii)** 4.5 N.

Question 6

Most candidates scored full marks in both parts **(i)** and **(ii)**. Common wrong answers in part **(i)** included 225 J from $\frac{1}{2} \times 50 \times 3^2$, 400 J from $\frac{1}{2} \times 50(7-3)^2$ and 7500 J from 'KE loss = PE gain = $50 \times 10 \times 15$'. Common wrong answers in part **(ii)** included 100 000 J from $50 \times 10 \times 200$ and 1000 J from 'PE gain = KE loss = $\frac{1}{2} \times 50(7^2 - 3^2)$' in part **(ii)**.

Part **(iii)** was less well attempted with many candidates omitting at least one of the three relevant components of the work done by the pulling force. Some candidates had just the resistance itself in linear combination with the two energy components, instead of the work done against the resistance.

The expectation of Examiners was that candidates would answer part **(iv)** by equating the answer found in part **(iii)** with $45 \times 200 \cos \alpha$. Indeed many candidates did just that, scoring 2 or all 3 of the available marks. However the majority of candidates used Newton's second law.

The equation thus derived contains four terms, one of which requires as a preliminary the calculation of the acceleration of the block. Not surprisingly the equation rarely contained all four terms, the component of weight and the mass-acceleration term being the most frequent absentees.

Another error using this method reflects a confusion for some candidates between the angle of inclination of the hill and the angle α. In the candidates' equation derived from Newton's second law the angle α is associated with the weight component instead of, or in some cases as well as, the pulling force of magnitude 45 N.

Answers: **(i)** 1000 J; **(ii)** 7500 J; **(iii)** 8000 J; **(iv)** 27.3.

Question 7

In part **(i)** the majority of candidates assumed that both particles travel with constant speed 1.3 ms^{-1}, notwithstanding that this assumption belies common sense. The notion is endorsed by many such candidates in assuming in part **(ii)** that each particle travels 3.25 m in the first 2.5 s. Another common error in part **(i)** was to take the common downward acceleration to be 10 ms^{-2}.

In part **(ii)** most candidates obtained either 1.04 ms^{-2} for the acceleration or 0.528 ms^{-2}. In each of these cases the answer is based on irrelevant use of the available data. In the former case the candidates simply took distance travelled in 2.5 s as 2.6×2.5 and thus found the acceleration from $6.5 = 1.3 \times 2.$…

In the latter case the candidates effectively assumed that Q comes to rest in a distance of 1.6 m a… found the acceleration from $0^2 = 1.3^2 - 2a(1.6)$.

The expectation of the Examiners was that candidates would use the idea that a particle moving freely on a smooth inclined plane has downward acceleration of $g\sin\theta$. Calculation of this acceleration is possible immediately from data in the question as $10\frac{1.6}{2.6\times2.5}$.

Part **(iii)** was the best attempted part of the question, many candidates scoring marks for a correct method, albeit based on an incorrect answer in part **(ii)** in almost all cases.

Answers: **(ii)** 2.46 ms^{-2}; **(iii)** 1.03 m.

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Paper 5

General comments

Candidates with a clear understanding of basic mechanical ideas made good progress on the paper and the majority of candidates had sufficient time to attempt all the questions

Most candidates gave answers to the required accuracy and not many used premature approximation. Nearly all candidates used the specified value of *g*.

The drawing of clear diagrams on the answer sheets would be a helpful aid to candidates in presenting their work. **Questions 6** and **7** would certainly have been clarified by a diagram.

Question 7 proved to be the most difficult question on the paper with very few candidates scoring maximum marks. Even the very good candidates often failed to clearly understand what was required in part **(iii)**.

Candidates scored highly on **Questions 2**, **3** and **5(i)** while candidates struggled with **Question 6**.

Comments on Individual Question

Question 1

This question proved to be a good start to the paper with many fully correct solutions. Sometimes '0.08' was misread as '0.8'. A few candidates considered only one string, getting $T\cos\theta = mg$ instead of $2T\cos\theta = mg$. A few candidates used Lami's Theorem.

In part **(ii)** Hooke's Law was generally well used. Errors usually occurred in trying to find the extension.

.A number of candidates attempted to use energy considerations which were not applicable in this question.

Answers: **(i)** 1.3 N; **(ii)** 15.6.

Question 2

(i) Weaker candidates used the wrong expression for the centre of mass of a cone, even though the correct expression is given in the List of Formulae.

(ii) The idea that the centre of mass is vertically above the lowest point of the base was used but too often poor trigonometry let candidates down.

(iii) This part was well done.

Answers: **(i)** 9.5 cm; **(ii)** 5.71 cm.

Question 3

(i) Most candidates set up two equations and went on to find L correctly. Some candid difficulty in eliminating T, making some simple manipulation error.

(ii) Too many unnecessarily long solutions appeared here. All that was expected was the use $v = r\omega$.

Answers: **(i)** 2.52; **(ii)** 3.18 m s^{-1}.

Question 4

(i) Newton's second law was often used but too frequently with a sign error i.e. $4 - 0.1v = 0.4\frac{dv}{dt}$. This equation suggests that the weight acts upwards! If these candidates had drawn a diagram perhaps the sign errors would have been eliminated.

(ii) Many recognised the need to separate the variables in order to integrate. Many errors occurred when trying to integrate. Logarithms of negative values sometimes appeared. The weaker candidates had difficulty in coping with algebra to separate the variables. Very few tried to use the equations of rectilinear motion.

Answer: **(ii)** 1.35.

Question 5

(i) This was well done by the majority of candidates.

(ii) Too many candidates could not take moments correctly and ended up with one side of the equation as a moment and the other side as just a force. i.e. $15 \times 0.22 = T\sin\theta$.

Answer: **(ii)** 30°.

Question 6

(i) A good diagram would have been very helpful to candidates. In part **(a)** $0.052 \times 10 \times d$ often seen, as was $\frac{0.8d^2}{4}$ in part **(b)** and $0.4 \times 0.052 \times 10$ in part **(c)**.

(ii) An attempt at a three term equation from their results in part **(i)** was usually seen but often the correct quadratic equation was not given since one, two or three wrong expressions were used from part **(i)**. On some occasions both answers were quoted when solving the quadratic and the smaller one was not rejected.

Answers: **(i)(a)** $0.48d$, **(b)** $0.2(d-2)^2$, **(c)** $0.08d$; **(ii)** 5.24.

Question 7

(i) Correct equations were often seen but then careless errors were made in finding v from the two equations or $R = \frac{v^2 \sin 2\theta}{g}$ was used with $R = 19.2$ instead of $R = 38.4$

(ii) Good candidates had no problems with this part. Weaker candidates tried to set up a quadratic equation in either x or t but made errors, usually getting a sign wrong.

(iii) This part proved too difficult for most candidates, with only a handful successfully reaching the final correct answer. Again the use of a clear diagram would have helped candidates.
A number of candidates incorrectly used $\sin\theta = \frac{12}{13}$, which is the value of $\sin\alpha$ in **Question 6**.

Answers: **(i)** 20; **(ii)** 32 m; **(iii)** 3.2 m.

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Paper 6

General Comments

This paper produced a wide range of marks. Many candidates failed to appreciate what the mode, mean and median represented in **Question 1**, which was meant to be an easy first question. There were a couple of difficult parts to the questions, but these were offset by some very easy parts thus enabling most candidates to make progress. Premature approximation was only a problem in **Question 2** and the normal distribution questions, where some candidates from certain Centres continued to work with 3 significant figures and thus did not achieve 3-figure accuracy in the final answer.

Comments on specific questions

Question 1

This question was the worst done in the paper. Many candidates thought they had to find all three of the mode, median and mean; and then say which other measures were not suitable e.g. stem-and-leaf. They clearly had no idea what 'central tendency' meant although this is a syllabus term and is frequently mentioned in textbooks Candidates would have gained a mark for mentioning the correct value for the median but most omitted the thousand at the end and just wrote 47.

Answers: median, 47 000, data have an outlier, mode is the lowest etc.

Question 2

This question was well done with many candidates scoring full marks.

Answers: **(i)** 0.4375; **(ii)** 0.3.

Question 3

There were some Centres where the majority of candidates could not attempt either of the normal distribution questions. Many candidates could attempt part **(ii)** but not part **(i)**. Of those who could attempt part **(i)** it was surprising to see the number of different *z*-values that were obtained: 1.644, 1.645, 1.646, 1.65 to mention a few. The tables give 1.645. The wrong *z*-value led to a premature approximation. The final part was only successfully attempted by the more able candidates although this sort of question has occurred many times before.

Answers: **(i)** 7.29; **(ii)** 0.136; **(iii)** 0.370.

Question 4

It was pleasing to see that most candidates knew about permutations and combinations. However, approximately half failed to connect the word 'arrangements' with permutations and used ${}_{17}C_{11}$ instead of ${}_{17}P_{11}$. Some successful candidates wrote 4.9×10^{11} and lost a mark for only writing the answer to 2 significant figures. Generally, if a candidate managed to answer part **(i)** successfully then they managed parts **(ii)** and **(iii)** successfully as well. Most candidates managed to pick up part marks for 5! seen and many gained full marks for part **(iii)**.

Answers: **(i)** 4.94×10^{11}; **(ii)** 79 833 600; **(iii)** 21.

Question 5

This question produced many excellent answers and also discriminated well. Most candidates kn frequency density meant and were able to convert it to a frequency successfully. However, many unable to appreciate that the final part needed to use the number of people over 25 years old and just us their **(ii)** divided by their **(iii)**, which scored no marks. Only the better candidates scored marks in this fina part.

Answers: **(i)** 30-35 years; **(ii)** 24; **(iii)** 110; **(iv)** 0.273.

Question 6

This question posed a few problems. The first two parts were generally well done but in part **(iii)** candidates became muddled as to which was the frequency and which was the variable. Candidates were able to obtain credit for 3 alternative tables, together with method marks for evaluating the mean and variance.

Answers: **(i)** 16; **(ii)** 8; **(iii)** matches 1, 2, 3, 4, 5 frequencies 16, 8, 4, 2, 2; **(iv)** mean 1.94, variance 1.43.

Question 7

Those candidates who knew their normal distribution scored high marks. A surprising number used the normal approximation to the binomial in both parts **(i)** and **(ii)**, scoring no marks in part **(i)**. Of those who recognised that part **(i)** was a question on the binomial distribution, only about half found the correct probability as being 1 – P(0, 1, 2). The mark scheme ensured that candidates who found any two binomial probabilities correctly could gain 2 marks for these, thus gaining part marks. A small minority found P(3, 4, 5, 6, 7, ...14) and an even smaller minority gained the correct answer using this method. Premature approximation to 3 significant figures in the working meant many lost the final accuracy mark.

In part **(ii)** the normal approximation to the binomial was well done. Mean and variance were correctly found and continuity corrections were much in evidence.

Answers: **(i)** 0.126; **(ii)** 0.281.

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Paper 7

Candidates found certain questions on this paper quite demanding, though there was plenty of opportunity, particularly in **Questions 1**, **2**, **3(ii)** and **5**, for the average candidate to gain marks. It is a pity that there is still confusion amongst candidates as to the three significant figure accuracy that is required. This has been mentioned in this report on numerous occasions, but still candidates lose valuable marks by seemingly not appreciating what is meant by significant figures, or perhaps writing, for example, an answer to three decimal places rather than to three significant figures. This is particularly important here, on a paper where many answers are probabilities. In **Question 6(iii)** where 0.0135 was the required answer to 3 significant figures, an answer of 0.014, as the only answer seen, would not have been accepted for the final answer mark. Candidates would be advised to over-specify their answers (i.e. write down several figures first) *before* doing a final answer, as less than 3.significant figures, as the only answer seen, will not score the marks as it is not to the required level of accuracy. It is also important that when candidates use any previously calculated figures in subsequent calculations they keep more than 3 significant figures when using that answer, otherwise accuracy will be lost for the next answer, again showing that it is a good habit for candidates to write more than 3.significant figures before giving their rounded answer, and then the more accurate answer is readily available for future use, preventing loss of marks.

Question 6 was poorly attempted, and it was disappointing that many candidates did not realise what was required for **Question 3(i)**. **Question 7(iii)**, which required an answer in the context of the question, was poorly attempted with a large number of candidates merely quoting a text book definition without relating it to the question. This has also been commented on in previous years.

The individual question summaries that follow include comments on how candidates performed along with common errors that were made. It should be remembered, when reading these comments, that there were some excellent scripts as well, where candidates gave exemplary solutions. Examiners noted that there did not appear to be a time issue on this paper.

Question 1

Most candidates used a correct formula, though there was some confusion seen between standard deviation and variance. The main error noted was in the value of z used, with 2.326 commonly seen instead of 2.576. Values of z between 2.574 to 2.579 were accepted, as some candidates obviously used the main Normal Distribution tables. It would, however, be quicker for candidates to use the small critical value table given in the list of formulae. Rounded values (e.g. 2.57 or 2.58) should not be used.

Answer: (98.8, 99.6).

Question 2

This question was, in general, well answered. Common errors were mainly in the calculation of the variance and included multiplying by 0.75 and 0.25 rather than 0.75^2 and 0.25^2, or by using the given standard deviations of 9.3 and 5.1 in the calculation rather than squaring these to use the variance. Many candidates did the correct calculation to find the variance, but then failed to square root this to give the standard deviation of the combined mark as requested.

Answers: 59.4, 7.09.

Question 3

Many candidates failed to realise what was required in part **(i)**. The three marks available were to the distribution of the sample means is normal, and to then to state its mean and variance. It was surp to find that many candidates, whilst unable to give a correct answer, or sometimes any answer, to part were able to gain full marks in part **(ii)**, thus *using* the correct distribution. Errors in part **(ii)** included failure to divide by 120 to obtain the variance of the sample means, or to incorrectly divide by 15 instead. Use of a continuity correction was occasionally seen.

Answers: **(i)** Normal with mean 6, variance 0.03; **(ii)** 0.282.

Question 4

Many candidates made a good attempt at this question, but often only found part of the required answer. The probability of ($D - W > 3$) *and* ($D - W < -3$), or equivalent, was required, and the majority of candidates found just one of these answers. Some candidates left their final answer as this, some attempted to double their one answer and very few considered both versions and combined correctly. Most candidates were able to find correctly the mean and variance of $D - W$, correctly standardise and then find the correct probability for either version, though use of a diagram could have helped some candidates at this point. Final answers of either 0.560 or 0.182 were extremely common. Weaker candidates considered $D - 3W$ or similar.

Answer: 0.742.

Question 5

This was a particularly well answered question, with even the weaker candidates scoring reasonably well, though the integration in part **(i)** proved to be a little challenging for some. Most candidates knew the method required to find k, but after correctly integrating and getting the correct expression of $\frac{4x^{k+1}}{k+1}$ many were then unable to correctly substitute the limits of 0 and 1. The most common error was to say that $4 \times 1^{k+1}$ was equal to 4^{k+1} thereby causing candidates to waste valuable time in using logarithms in a now incorrect attempt to show that k was equal to 3. Part **(ii)** was well attempted, with most candidates correctly showing that the mean was 0.8, and it was pleasing to note that few candidates made the, usually common, error of forgetting to subtract $(E(X))^2$ in their expression for the variance of X. There was some confusion in parts **(iii)** and **(iv)** between the upper and lower quartile, with some candidates calculating them the wrong way round. Other errors in method included using the limits in the integration as 0.75 or 0.25.

Answers: **(ii)** 0.0267; **(iii)** 0.931; **(iv)** 0.223.

Question 6

This question was poorly attempted. Whilst some candidates appreciated that a Poisson distribution should be used, many used the wrong parameters throughout the question and failed to combine probabilities correctly. Incorrect calculations in part **(ii)** such as $e^{-0.6} \times e^{-0.3}$ or $e^{-0.6} \times 0.03$ rather than $e^{-0.6} \times 0.97$ were often seen, though part **(iii)** was better attempted. A Poisson approximation to the binomial in part **(iv)** was required, but this was seen rarely, with most candidates assuming a normal approximation or not using an approximation at all and using the binomial. As the answer to part **(iii)** was required in part **(iv)** it was important here to use the answer to part **(iii)** to more than 3.significant figures as mentioned earlier.

Answers: **(i)** 0.122; **(ii)** 0.532; **(iii)** 0.0135; **(iv)** 0.229.

Question 7

It was disappointing to see such a variety of elementary errors in part **(i)** of this question. Incorre such as n = 49, $\sum fx$ = 1755 or 371 were often seen, and there were many formula mistakes, partic use of the formula for the biased variance rather than unbiased, and confusion between $\frac{\sum x^2}{n}$ and $\frac{\bar{x}^2}{n}$.

In part **(ii)** many candidates correctly stated the alternative hypothesis and made a good attempt to find the test statistic. The main cause of loss of marks here was in the comparison of the test statistic with the critical value of –1.645 (or equivalent comparison). Many candidates did not show this comparison and merely stated a conclusion. This is not sufficient to gain full marks; the justification of the conclusion must be stated. Part **(iii)** was not well attempted, with many candidates merely stating text book definitions, and not giving enough detail on their diagram (i.e. showing the mean of 7.2 or labelling the 5%)

Answers: **(i)** 6.53, 2.87; **(iii)** Say there is a reduction in the number of cars caught speeding when there is not.